B.Sc Part - I

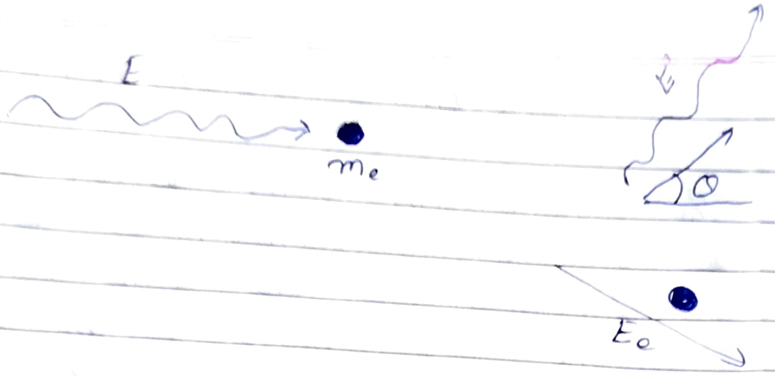
COMPTON SCATTERING

The momentum four vector of light can be written in consonance with the interpretation of the time part as energy, and the space part as the linear momentum. Thus invoking quantum mechanics we have the momentum four vector for light

$$K_{light} = \left(\frac{h\nu}{c}, hK\right) \quad \text{--- (1)}$$

Scattering of a massive particle by light is referred to as compton scattering. A simple scenario is shown in fig. which a photon of energy $E = h\nu$ collides with a electron of mass m_e. After collision the rest mass of the electron stays the same, although it has acquired some kinetic energy. The wavelength of the scattered photon is related to the angle of scattering. It is this relation that we try to find here. For convenience we have chosen a reference frame in which the particle is initially at rest. Besides the scattering the assumed to happen in the x-y plane. The momentum four vectors for light and the particle before and after the collision are respectively,

Before $$\left(\frac{h\nu}{c}, \frac{h\nu}{c}, 0\right), \left(m_e c, 0, 0\right) \quad \text{--- (2)}$$



Scattering of a electron, initially at rest, by a photon of energy $E = h\nu$. The change in frequency of the photon depends on the angle of scattering θ.

After

$$\left(\frac{h\nu'}{c}, \frac{h\nu'}{c}\cos\theta, \frac{h\nu'}{c}\sin\theta\right),$$

$$\left(\frac{E_e}{c}, P_{ex}, P_{ey}\right) \quad \text{--- (3)}$$

where

$$E_e^2 - (P_{ex}^2 + P_{ey}^2)c^2 = m_e^2 c^4 \quad \text{--- (4)}$$

and ν' is the frequency of the scattered photon. Conservation of four-momentum gives

$$h\nu + m_e c^2 = h\nu' + E_e \quad \text{--- (5)}$$

$$h\nu = h\nu'\cos\theta + P_{ex}c, \quad \text{--- (6)}$$

$$h\nu'\sin\theta = P_{ey}c \quad \text{--- (7)}$$

substituting eqn (5) & (7) in eqn (4) we have

$$m_e^2 c^4 = (h\nu + m_e c^2 - h\nu')^2 - (h\nu - h\nu'\cos\theta)^2 - h\nu'^2\sin^2\theta \quad \text{--- (8)}$$

ultimately, this reduces to

$$0 = -(\nu - \nu')m_e c^2 + h\nu\nu' - h\nu\nu'\cos\theta \quad \text{—} \quad (9)$$

Substituting $\nu = c/\lambda$ and $\nu' = \frac{c}{\lambda'}$ in eq^n (9) we get

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \quad \text{—} \quad (10)$$

Eq^n (10) gives the desired relation between change in wavelength and the angle of scattering. The quantity $\frac{h}{(m_e c)}$ is the compton wavelength of electron and is roughly equal to 2.43×10^{-12} m. The angle of scattering θ may vary from zero. (no scattering or forward scattering) to $180°$).